NOLTA, IEICE

Paper

Error-correcting scheme based on chaotic dynamics and its performance for noncoherent chaos communications

Shintaro Arai [1 a)], *Yoshifumi Nishio* [2 b)], and *Takaya Yamazato* [3 c)]

[1] *ITS Lab., Faculty of Engineering, Aichi University of Technology 50-2 Manori, Nishihasama-cho Gamagori, Aichi 443-0047, Japan*

[2] *Dept. of Electrical and Electronic Engineering, Tokushima University 2-1 Minami-Josanjima, Tokushima 770-8506, Japan*

[3] *EcoTopia Science Institute, Nagoya University C3-1 Furo-cho, Chikusa-ku, Nagoya 464-8063, Japan*

a) *arai-its@aut.ac.jp*
b) *nishio@ee.tokushima-u.ac.jp*
c) *yamazato@nuee.nagoya-u.ac.jp*

Abstract: This paper proposes a novel error-correcting scheme using chaotic dynamics for noncoherent chaos communications. In our proposed system, two successive chaotic sequences are generated from the same chaotic map; the second sequence is generated with an initial value which is the last value of the first sequence. In this case, successive chaotic sequences having the same chaotic dynamics are created. This feature gives the receiver additional information to correctly recover the information data and thus improves the bit error performance of the receiver. As results of the computer simulation, we confirm that the advantage gained in BER performance of the proposed error-correcting method is about 1–1.5 dB compared to conventional method. In addition, we achieve that the proposed error-correcting scheme is performed without the new additional redundancy code by using the chaotic dynamics.

Key Words: chaos, noncoherent chaos communications, error-correcting scheme, chaotic dynamics

1. Introduction

Chaos has received a great deal of attention in the past few years from a variety of researchers, including mathematicians, physicists and engineers [1, 2]. The research direction has been transferring from finding the evidence of chaos existence into applications and deep theoretical study in recent years. Chaotic sequences obtained from a certain class of difference equations are nonperiodic and

[Received: January 11, 2010. Revised: May 8, 2010. Published: October 1, 2010.]
[DOI: 10.1588/nolta.1.196]
Nonlinear Theory and Its Applications, IEICE, vol. 1, no. 1, pp. 196–206

sensitive to initial conditions, and it is difficult to predict their future behavior from past observations. It was also confirmed that even simple one-dimensional maps can generate chaotic sequences. Since it is becoming comparatively easy to implement chaotic system, many researchers in the field of nonlinear circuits and systems have largely been interested for the development of chaos applications.

Chaos communication systems are one of interesting topics in the field of engineering chaos [3–12]. Especially, many researchers have focused on the development of noncoherent detections which do not need to use basis signals (unmodulated carriers) for demodulation at a receiver. In standard communication systems, which are categorized as coherent detections, basis signals need to be reproduced for demodulation at the receiver. Thus, it is said that the standard communication systems are difficult to operate the noncoherent detection. On the other hands, the noncoherent detection using chaos can demodulate the information without basis signals since chaos and chaotic sequences have unique features itself. Therefore, we consider that the noncoherent detection is unique detection method using chaos. Differential chaos shift keying (DCSK) [3] and the optimal receiver [4] are well-known typical noncoherent systems. In this study, we assume the noncoherent detection as chaos communication systems and focus on the optimal receiver.

The optimal receiver calculates the probability density function (PDF) between the received signal and the chaotic maps used on the transmitting side and detects the information symbol by choosing the larger probability. However, the optimal receiver suffers from high computational complexity. Also, using long chaotic sequence decreases the performance of the optimal receiver. Thus, it is important to develop a receiver with performance equivalent to or similar to the optimal receiver using more efficient algorithms, i.e., a suboptimal receiver.

In our previous research, we proposed the suboptimal receiver using the shortest distance approximation [13]. Instead of calculating the PDF, the proposed suboptimal receiver approximates the PDF by calculating shortest distances between the received signal and the chaotic maps and performs detection of the transmitted symbol. As results of the computer simulations, we confirmed the validity of the proposed suboptimal receiver as an approximation method of the optimal receiver.

Note that the suboptimal receiver shows a slight performance loss to the optimal receiver. In other words, the detection characteristic of the suboptimal receiver is not superior to the optimal receiver. However, we expect to improve the performance of the suboptimal receiver by using characteristics of chaos. Because we investigated and analyzed the influence on chaos communications systems by the chaotic dynamics and its behavior previously [14, 15] and confirmed that the chaotic dynamics affect the performance of chaos communication systems greatly. Therefore, we consider that it is important to develop and advance communication systems based on characteristics of chaos.

For advancing the noncoherent chaos communication system in this paper, we focus on the chaotic dynamics, which is one of characteristics of chaos, and propose a novel error-correcting scheme based on the chaotic dynamics. In standard noncoherent chaos communication systems, a binary data bit is modulated in a transmitter by a chaotic sequence of chosen length; the bit is demodulated at the receiver using the modulated sequence and the transmitted unmodulated corresponding sequence. The demodulation of each bit is performed only with the chaotic sequence associated with that bit. We have paid attention to the successive chaotic sequences between symbols which are scarcely used for the demodulation in standard chaos communications. Namely, we utilize the chaotic dynamics among multiple symbols for the error correction.

In our proposed system, two successive chaotic sequences are generated from the same chaotic map; the second sequence is generated with an initial value which is the last value of the first sequence. In this case, successive chaotic sequences having the same chaotic dynamics are created. This feature gives the receiver additional information to correctly recover the information data and thus improves the bit error performance of the receiver. Further, the proposed error-correcting scheme is operated without a redundancy bit sequence referred to perform an error correction in standard communication systems. Note that the chaotic sequence already has a kind of redundancy since chaos is well-known as the wideband signal and is expressed as multiple bits. Thus, the proposed method does not require new additional redundancy bit sequence since the error correction applies the chaotic dynamics having the transmitted signal blocks. We carry out computer simulations and evaluate the bit error rate (BER)

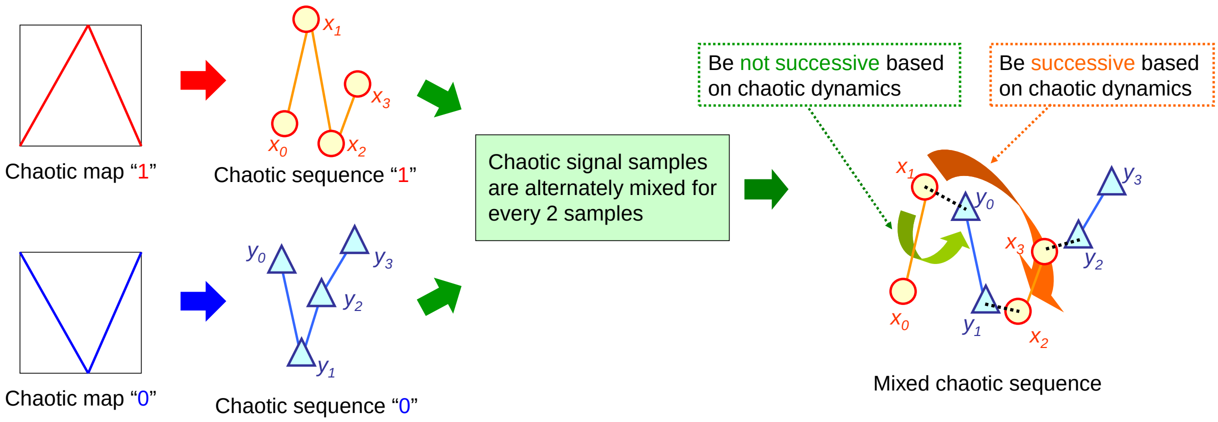


Fig. 1. Concept of proposed error-correcting method.

performance of the proposed method.

In Sect. 2, a concept of our proposed error-correcting scheme is introduced. In Sect. 3, a system model and an operation of our proposed error-correcting scheme are described. In Sect. 4, we discuss computer simulations that were performed to evaluate its performance. Finally, we conclude this paper in Sect. 5.

2. Concept of proposed error-correcting scheme

In this section, we explain a concept of the proposed error-correcting scheme using Fig. 1. Our proposed error-correcting scheme applies the chaotic dynamics. A left side of Fig. 1 shows 2 different chaotic maps and sequences. In this figure, x_i and y_i $(i = 0, 1, \cdots)$ denote a sample of the chaotic sequence "1" and "0" generated by the chaotic map "1" and "0" respectively.

Here, let us consider the case that the signal samples of these sequences are alternately mixed for every 2 samples, as shown in a right side of Fig. 1. In this mixed chaotic sequence, the chaotic dynamics of some successive samples is lost. For instance, since y_0 is not generated by x_1, x_1 and y_0 are not successive based on the identical chaotic dynamics. Namely, the mixed chaotic sequence becomes a train of sequence which is not successive based on the chaotic dynamics.

However, we consider that the loss of the chaotic dynamics can be applied for communication systems. Now, let us assume that we do not know the rule of how to mix 2 different chaotic sequences. Although we do not know this rule, we know that the chaotic signal samples before mixing are successive according to the chaotic dynamics. For example, x_1 influences generation x_2. Namely, even if the rule of how to mix the sequences is never known, we reconstruct the original chaotic sequences by analyzing the chaotic dynamics of samples of the mixed chaotic sequence.

We use this feature as an error correction for noncoherent chaos communications. This feature gives the receiver additional information to correctly recover the information data and thus improves the bit error performance of the receiver. In the next section, we describe a system model and an operation of the proposed error-correcting scheme in detail.

3. System model with proposed error-correcting scheme

We consider the discrete-time binary CSK communication system with the error correcting, as shown in Fig. 2. Detail of each block is described below.

3.1 Transmitter

In the transmitter, binary data are encoded by chaotic sequences generated by a chaotic map. In this study, we use a skew tent map which is one of simple chaotic maps, and it is described by Eq. (1)

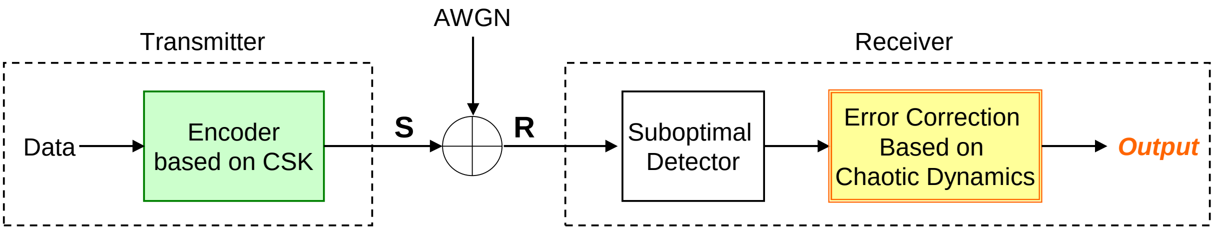


Fig. 2. Block Diagram of Discrete-Time Binary CSK Communication System.

$$x_{i+1} = \begin{cases} \dfrac{2x_i + 1 - a}{1 + a} & (-1 \leq x_i \leq a) \\ \dfrac{-2x_i + 1 + a}{1 - a} & (a < x_i \leq 1) \end{cases} \tag{1}$$

where a denotes a position of the top of the skew tent map. Our encoder is designed based on Chaos Shift Keying (CSK) which is a digital modulation system using chaos. Figure 3 shows our encoder for our error-correcting scheme. To perform the error correction at the receiver, K information bit are transmitted as K signal blocks $(0, 1, \cdots, j, \cdots, K-1)$. The encoder selects a chaotic signal generator according to the symbol. Here, we use "f" to refer to the function of the skew tent map of Eq. (1). If the symbol "1" is sent, f is used, and if "0" is sent, g $(= -f)$ is used. Thus, the signal vector $\mathbf{S_j}$ is different for each symbol.

When the symbol "1" is sent

$$\mathbf{S_j} = (x_\alpha,\ f^{(1)}(x_\alpha), \cdots, f^{(i)}(x_\alpha), \cdots, f^{(N-1)}(x_\alpha)). \tag{2}$$

When the symbol "0" is sent

$$\mathbf{S_j} = (y_\alpha,\ g^{(1)}(y_\alpha), \cdots, g^{(i)}(y_\alpha), \cdots, g^{(N-1)}(y_\alpha)). \tag{3}$$

where i is the iteration of f or g, $\alpha = N \times j$, x_j or y_j denotes the initial value of the j-th symbol = "1" or "0" respectively, N is the chaotic sequence length per 1 bit. When K bit data is transmitted, the amount of the data becomes $K \times N$. Thus, the amount of the data per bit of the proposed method becomes the same as the standard CSK.

An initial value is chosen at random when beginning to make signal blocks and is different in each chaotic signal generator. In addition, the j-th sequence is generated from the initial value which is the end value of the former sequence with same symbol of j-th bit. As an example, we assume $N = 2$, $K = 4$ and the data are (1, 0, 0, 1) shown in Fig. 3. In this case, the transmitted signal vector $\mathbf{S}$ is given as follows.

$$\begin{aligned} \mathbf{S} &= (\mathbf{S_0}, \mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3}) \\ &= (x_0, x_1, y_0, y_1, y_2, y_3, x_2, x_3) \\ &= (s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7). \end{aligned} \tag{4}$$

As one can see, the initial value of the 4th symbol and 3rd symbol is generated by the end value of 1st symbol and 2nd symbol, respectively.

In this scheme, the transmitted signal blocks do not follow a redundancy blocks referred to perform an error correction in standard communication systems. Note that the chaotic sequence already has a kind of redundancy since chaos is well-known as the wideband signal and is expressed as multiple bits. We focus on the redundancy of chaos as the important characteristic of chaos, i.e. the chaotic dynamics. Thus, the proposed method does not require new additional redundancy bit sequences since the error correction applies the chaotic dynamics having the transmitted signal blocks.

3.2 Channel and noise

The channel distorts the signal and corrupts it by noise. In this study, noise of the channel is assumed to be the additive white Gaussian noise (AWGN). Thus, the received signal block is given by $\mathbf{R} = (r_0,\ r_1, \cdots,\ r_{KN-1}) = \mathbf{S} + \text{AWGN}$.

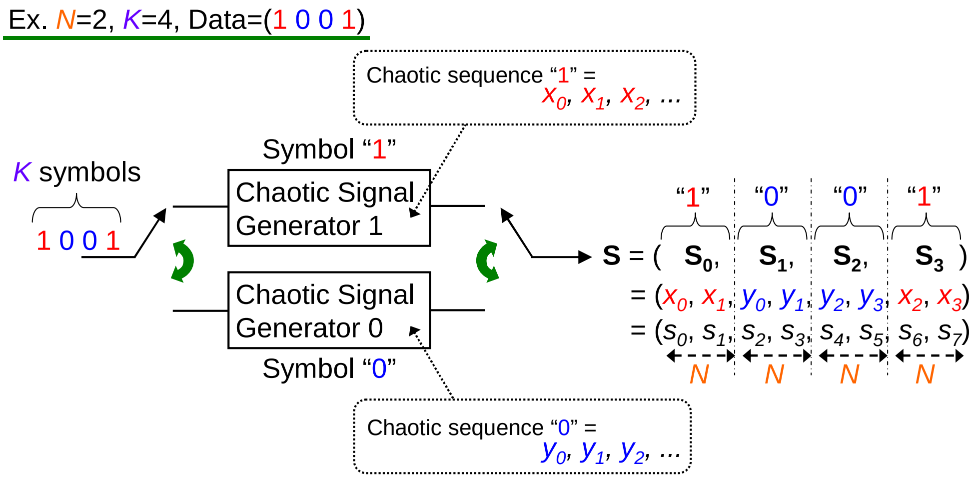


Fig. 3. Encoder based on CSK for error correction.

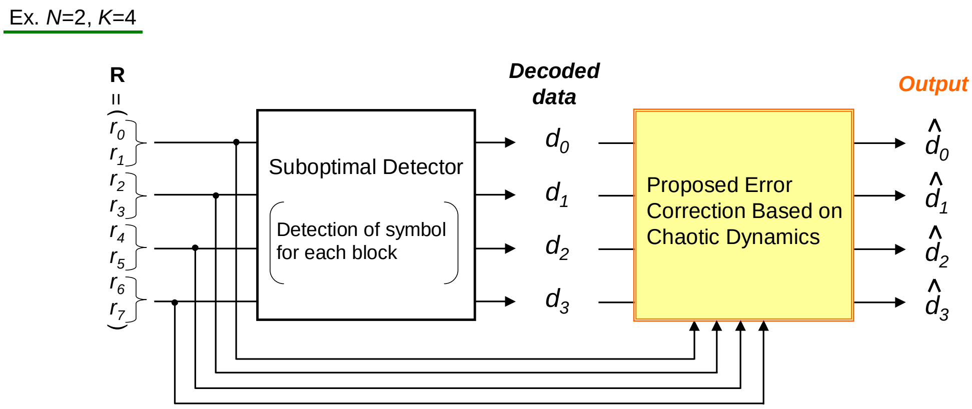


Fig. 4. Operation of proposed error-correcting method.

3.3 Noncoherent receiver with proposed error correction

The receiver recovers the transmitted signal blocks from the received signal blocks and demodulates the information symbol. Also, the receiver performs the error correction in this study. Since we consider the noncoherent receiver, the receiver memorizes the chaotic map used for the modulation at the transmitter. However, the receiver never knows the initial value of chaos in the transmitter. Our proposed error-correcting method consists of the suboptimal detector and the error correction based on chaotic dynamics, as shown in Fig. 4. First of all, the receiver performs the noncoherent detection for each received block and demodulates each symbol. In this study, we apply our suboptimal detection algorithm as the noncoherent detection [13]. After demodulation of each symbol, the receiver performs the error-correcting scheme. Before explaining the operation of the proposed error correction, we describe the operation of our suboptimal detector to be the basis for the proposed scheme.

Suboptimal detector

Our suboptimal detector calculates the shortest distance between $\mathbf{R}'_i$ and N_d-dimensional space made from N_d successive chaotic signals generated by the skew tent map, $(N_d : 2, 3, \cdots)$, and outputs the sum of the distance, which achieves the minimum shortest distance within the set of l, between $i = 0$ and $N - N_d$. Here, $\mathbf{R}'_i$ is the N_d successive signals beginning with i with in the received signals, and l is the number of the straight lines in the N_d-dimensional space, which are defined as

$$\mathbf{R}'_i = (r_{\alpha+i}, r_{\alpha+i+1}, ..., r_{\alpha+i+N_d-1}). \tag{5}$$

$$l = 2^{N_d-1}. \tag{6}$$

When two symbols are transmitted, two kinds of the N_d-dimensional space corresponding to each symbol are made. Let us consider the case of Symbol "1". For calculating the shortest distance, we

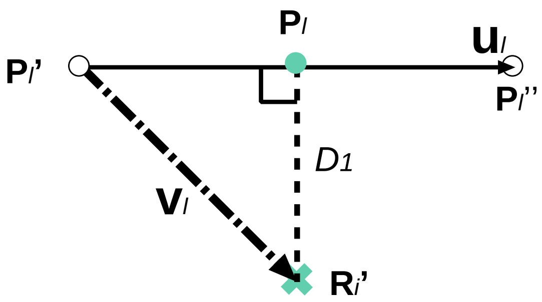


Fig. 5. Calculation of shortest distance.

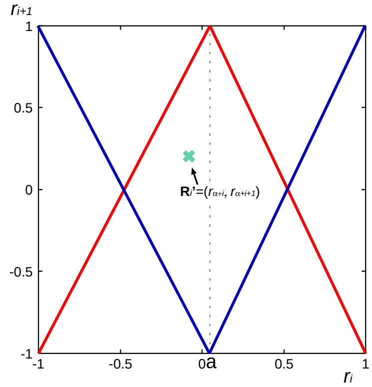


Fig. 6. N_d =2-dimensional space.

find the closest point $\mathbf{P}_l$ between $\mathbf{R'}_i$ and the lth line using the scalar product of the vector. When both edges of the lth line are defined as $\mathbf{P'}_l$ and $\mathbf{P''}_l$ shown in Fig. 5, the closest point $\mathbf{P}_l$ is calculated by the following equation.

$$\mathbf{P}_l = \left(p_0^{(l)}, p_1^{(l)}, \cdots, p_{N_d-1}^{(l)}\right) = (\mathbf{u}_l \cdot \mathbf{v}_l)\,\mathbf{u}_l + \mathbf{P'}_l. \tag{7}$$

where

$$\text{unit vector } \mathbf{u}_l = \frac{\mathbf{P''}_l - \mathbf{P'}_l}{||\mathbf{P''}_l - \mathbf{P'}_l||}. \tag{8}$$

$$\mathbf{v}_l = \mathbf{R'}_i - \mathbf{P'}_l. \tag{9}$$

In the same ways, we can find the closest point $\mathbf{Q}_l$ between $\mathbf{R'}_i$ and the lth line of the space of Symbol "0".

Then, the above operations are expressed as

$$\sum D_1 = \sum_{i=0}^{N-N_d} \min_l ||\mathbf{P}_l - \mathbf{R'}_i||. \tag{10}$$

$$\sum D_0 = \sum_{i=0}^{N-N_d} \min_l ||\mathbf{Q}_l - \mathbf{R'}_i||. \tag{11}$$

Finally, we decide the decoded symbol as 1 (or 0) for $\sum D_1 < \sum D_0$ (or $\sum D_1 > \sum D_0$).

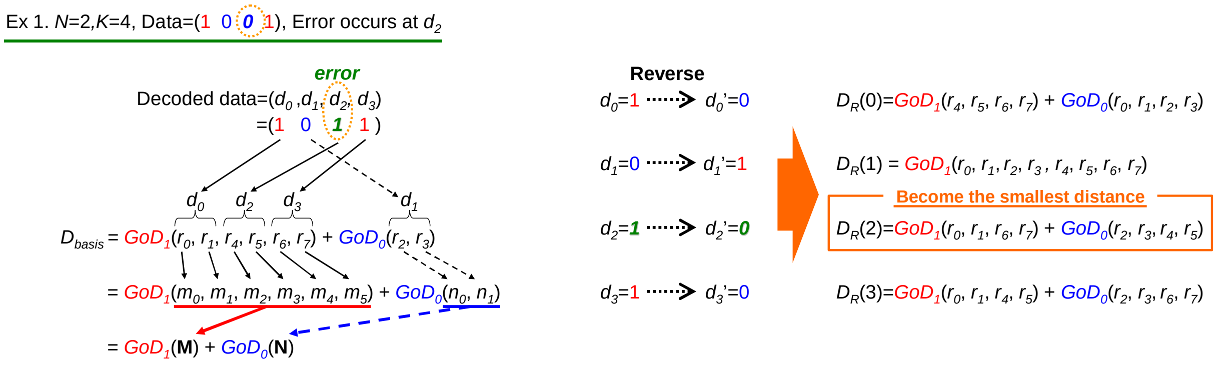
As an example, we explain the case of $N = 3, N_d = 2$. Figure 6 shows the N_d =2-dimensional space of the skew tent map whose coordinates correspond to the two successive received signals $\mathbf{R'}_i = (r_{\alpha+i}, r_{\alpha+i+1})$. In this case, we can calculate $\sum D_1$ and $\sum D_0$, as follows.

$$\begin{aligned}\sum D_1 &= \min_l ||\mathbf{P}_l - \mathbf{R'}_0|| + \min_l ||\mathbf{P}_l - \mathbf{R'}_1|| \\ &= \min_l \left\{\sqrt{\left(p_0^{(l)} - r_0\right)^2 + \left(p_1^{(l)} - r_1\right)^2}\right\} + \min_l \left\{\sqrt{\left(p_0^{(l)} - r_1\right)^2 + \left(p_1^{(l)} - r_2\right)^2}\right\}. \end{aligned} \tag{12}$$

$$\begin{aligned}\sum D_0 &= \min_l ||\mathbf{Q}_l - \mathbf{R'}_0|| + \min_l ||\mathbf{Q}_l - \mathbf{R'}_1|| \\ &= \min_l \left\{\sqrt{\left(q_0^{(l)} - r_0\right)^2 + \left(q_1^{(l)} - r_1\right)^2}\right\} + \min_l \left\{\sqrt{\left(q_0^{(l)} - r_1\right)^2 + \left(q_1^{(l)} - r_2\right)^2}\right\}. \end{aligned} \tag{13}$$

3.4 Operation of proposed error-correcting method

After demodulation of each symbol, the receiver performs the error-correcting method. For error correction without the new additional redundancy code, the receiver uses the received signal samples again. For ease of explanation, we use Fig. 7 and explain an operation of the proposed error-correcting



(a) (b)

Fig. 7. Analysis of Chaotic Dynamics using Suboptimal Detection for 1 bit correlation: (a) Calculation of D_{basis}, (b) Calculation of $D_R(j)$.

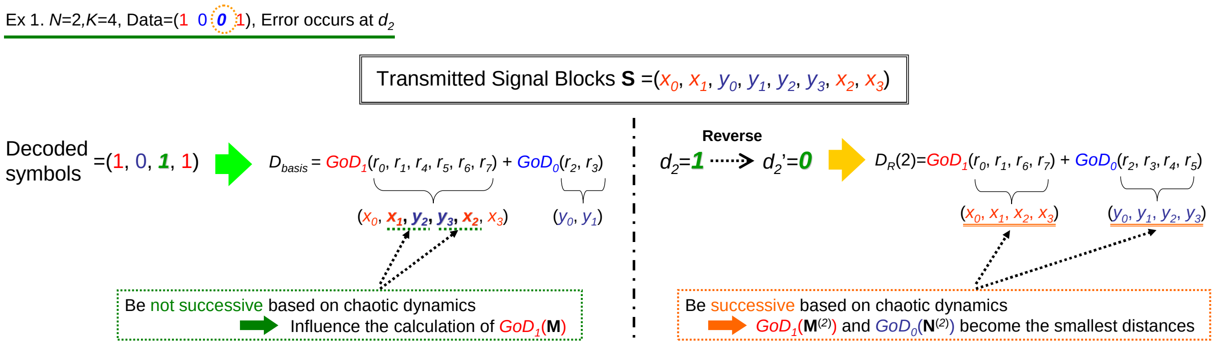


Fig. 8. Reason why $D_R(2)$ becomes the smallest distance.

scheme. Here, we use same assumption in the explanation of the encoder (Fig. 3). Also, we assume that the detection error has occurred at the 2nd symbol (d_2), namely the case of 1 bit correction.

First of all, the receiver sorts the received signal samples based on decoded symbols, as shown in Fig. 7(a). Here, we define the sequence sorted samples based on decoded symbol "1" (or "0") as $\mathbf{M} = (m_0, m_1, \cdots, m_{C_1-1})$ (or $\mathbf{N} = (n_0, n_1, \cdots, n_{C_0-1})$), where C_1 and C_0 are the total of $\mathbf{M}$ and $\mathbf{N}$, respectively. Next, the receiver analyzes the chaotic dynamics of $\mathbf{M}$ and $\mathbf{N}$. If the receiver can detects symbols and sorts blocks correctly, we can obtain two successive chaotic sequences based on the chaotic dynamics. However, if the detection error occurs when the receiver detects symbols, the sorted sequence mixes two chaotic sequences which differ in the chaotic dynamics. We focus on these characteristics of chaos for the error-correcting. For analyzing the chaotic dynamics, the receiver applies our suboptimal detection algorithm, i.e., the calculation of the shortest distance between the chaotic maps and two sorted received sequences. Thus, we define a reference distance D_{basis} as follows.

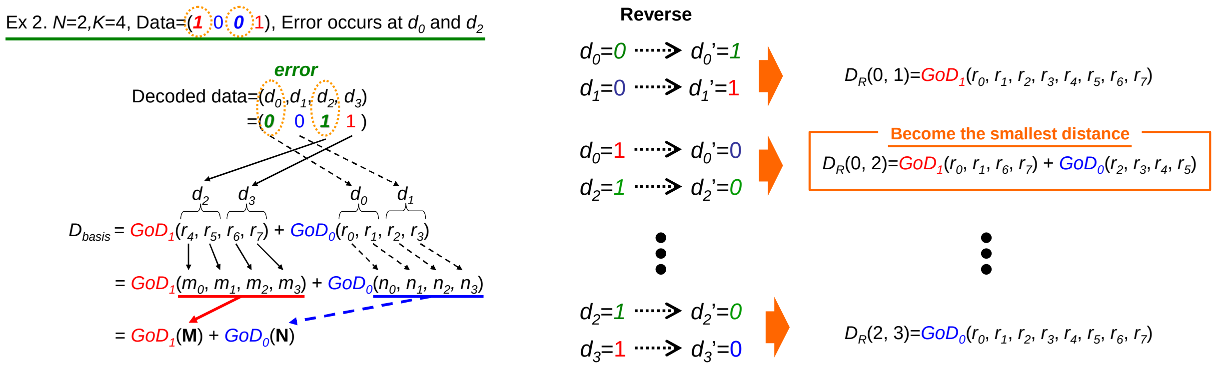
$$D_{basis} = GoD_1(\mathbf{M}) + GoD_0(\mathbf{N}). \tag{14}$$

where $GoD_1(\mathbf{M})$ (or $GoD_0(\mathbf{N})$), which is short for "Group of D_1 (or D_0)", is the shortest distance between $\mathbf{M}$ (or $\mathbf{N}$) and the N_d-dimensional space of Symbol "1" (or "0") defined as

$$GoD_1(\mathbf{M}) = \sum_{i=0}^{C_1-N_d} \min_l ||\mathbf{P}_l - \mathbf{M}'_i||. \tag{15}$$

$$GoD_0(\mathbf{N}) = \sum_{i=0}^{C_0-N_d} \min_l ||\mathbf{Q}_l - \mathbf{N}'_i||. \tag{16}$$

Next, we calculate the distance $D_R(j)$ for comparing with D_{basis}, where the subscript R means initial character of "Reverse". This function means the shortest distance between sorted sequence when the j-th decoded symbol is reversed and the chaotic map corresponding to their sequences. Namely, we assume the detection error occurs at the j-th symbol and calculate $D_R(j)$ as follows.



(a) (b)

Fig. 9. Analysis of Chaotic Dynamics using Suboptimal Detection for 2 bit correction: (a) Calculation of D_{basis}, (b) Calculation of $D_R(j,k)$.

$$D_R(j) = GoD_1(\mathbf{M}^{(j)}) + GoD_0(\mathbf{N}^{(j)}). \tag{17}$$

where $\mathbf{M}^{(j)}$ and $\mathbf{N}^{(j)}$ denote sorted sequences when the j-th decoded symbol is reversed. If the receiver can detects symbols and sorts blocks correctly in the first step, $D_R(j)$ becomes lager values as compared with D_{basis}. On the other hands, if the detection error occurs, some one of $D_R(0)$–$D_R(K-1)$ become smaller as compared with D_{basis}. For instance, Fig. 7(b) shows conceivable combinations of sorted sequences and calculates $D_R(j)$ when the detection error occurs at the 2nd symbol. In this case, $D_R(2)$ becomes the smallest distance as compared with D_{basis} and other $D_R(j)$. Thus, in this example, the receiver can determine that the detection error occurs at the 2nd symbol.

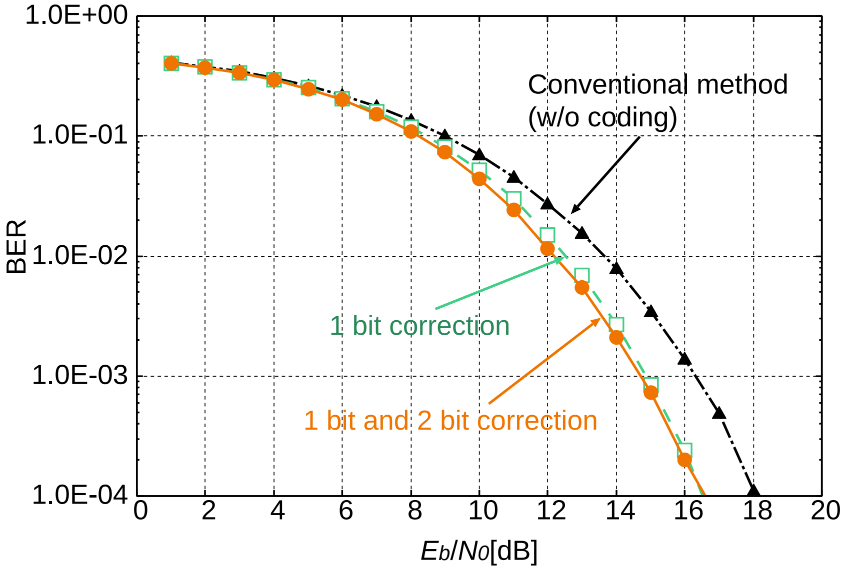
The reason for this is that the receiver reconstructs the original chaotic sequences of symbol "1" and "0" from the received signal blocks. Figure 8 shows the transmitted signal blocks of this example and sequences sorted according to the rule of calculating D_{basis} and $D_R(2)$. As one can see, the sequence of decoded symbol "1" for calculating D_{basis} is not successive based on the identical chaotic dynamics because y_2 and x_2 are not generated by x_1 and y_3, respectively. In other words, since the received signal samples generated by the different chaotic map are successive, the influence between these symbols appears in the calculation result of the shortest distance. Thus, these samples influence the calculation of $GoD_1(\mathbf{M})$ of D_{basis}. On the other hands, 2 sequences for calculating $D_R(2)$ become the successive sequences based on each chaotic dynamics because the receiver reverses the false symbol and reconstructs the original sequence according to the reversed symbol. Hence, $GoD_1(\mathbf{M}^{(2)})$ and $GoD_0(\mathbf{N}^{(2)})$ of $D_R(2)$ become the smallest distances as compared with those of D_{basis} and other $D_R(j)$. Therefore, the receiver selects the smallest distance from D_{basis} and $D_R(j)$ and corrects an error.

Although we consider the case of the 1 bit correction in this assumption, the case of 2 or more bit can be also performed in the same way. As a difference between the 1 bit correction and the 2 or more bit correction, the number of the symbols reversed at once increases according to the number of bit for the correction. Figure 9 shows an example of an analysis for 2 bit correction. In the calculation of $D_R(*)$ for 2 bit correction, we assume the detection errors occur at the j-th and the k-th symbol simultaneously. Thus, we calculate $D_R(j,k)$ as follows.

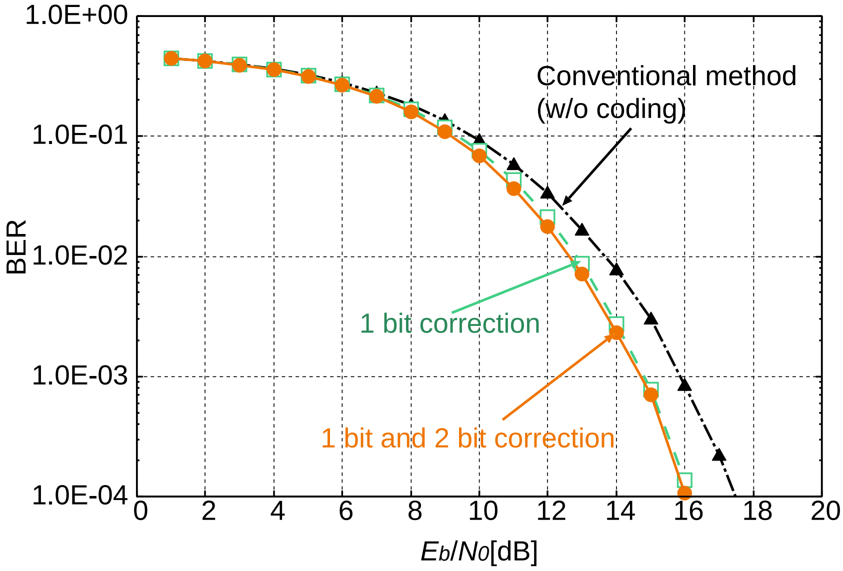
$$D_R(j,k) = GoD_1(\mathbf{M}^{(j,k)}) + GoD_0(\mathbf{N}^{(j,k)}). \tag{18}$$

4. Evaluation of proposed error-correcting method

In this section, we evaluate the performance of the proposed error-correcting method by computer simulations. The simulation conditions are as follows. In the transmitting side, we assume $K = 32$. The parameter of the skew tent map is fixed as $a = 0.05$. For calculation of the shortest distance, we use 4-dimensional space, namely $N_d = 4$. Moreover, we perform the proposed error-correcting scheme



(a) $N = 4$.



(b) $N = 8$.

Fig. 10. BER vs. E_b/N_0 ($K = 32$ and $N_d = 4$).

until 2 bit error correction. Based on these conditions, we iterate the simulation 10,000 times and calculate BER performance.

Figures 10(a) and (b) show the BERs versus E_b/N_0 for $N = 4$ and $N = 8$, respectively. We plot the performance of the proposed error-correcting scheme and the performance of the conventional method, namely, the performance without the error-correcting scheme in Figs. 10(a) and (b). From these figures, we can confirm that the advantage gained in BER performance of the proposed error-correcting method is about 1–1.5 dB compared to conventional method. Namely, we achieve that the error-correcting scheme is performed without new additional redundancy blocks by using the chaotic dynamics. However, the performance of 1 bit and 2 bit correction is only slightly better than that of 1 bit correction. In general, it is well-known that the percentage of 1 bit error is statistically larger than that of 2 bit error. Thus, the effect of 1 bit correction by using the proposed error-correcting method on BER performance is also larger than that of its 2 bit correction. This problem might be solved by increasing the number of bit for the correction, On the other hand, since the number of conceivable combinations also increases, more computation time is needed. Therefore, it can be said that the 1 bit correction is currently more accessible than that of 1 bit and 2 bit correction.

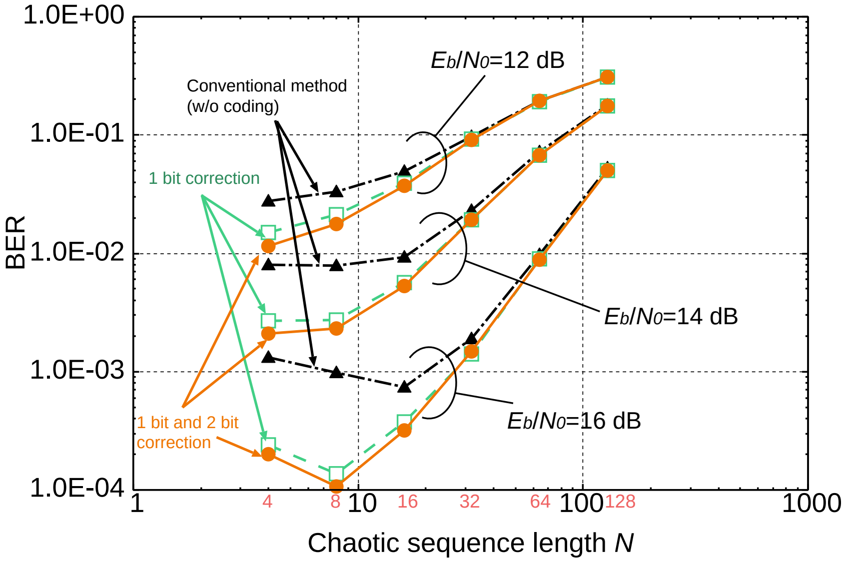


Fig. 11. BER vs. Chaotic sequence length N ($K = 32$ and $N_d = 4$).

Figure 11 plots the BERs versus the chaotic sequence length N for different E_b/N_0 values. In the region of small N, we can confirm the effect of the proposed scheme. Especially, when $E_b/N_0 = 16$, the BER of $N = 8$ improves from 1×10^{-3} to 1×10^{-4} using the proposed error-correcting scheme. However, in the region of large N, the effect of the proposed scheme decreases. The reason for this is that central samples of each sequence influence the error correction when N becomes large. In the proposed scheme, we assume the detection error occurs at the arbitrary symbol and sort the received sequences. In addition, we calculate $D_R(j)$ and $D_R(j,k)$ using the sorted sequences. Note that the joint of the sorted sequence most influences the calculation of $D_R(j)$ and $D_R(j,k)$. In other words, since central samples of each sequence are successive based on the chaotic dynamics, the influence of the chaotic dynamics having central samples is stronger than that of the joint of the sorted sequence in the case of large N, Thus we consider that the effect of the proposed scheme decreases according to increase in N. Therefore, to design the error-correcting scheme with central samples of chaotic sequences is our future problem.

5. Conclusions

We have proposed the novel error-correcting scheme for noncoherent chaos communications. We have considered that the chaotic dynamics can be applied as additional information to correctly recover the information data. Further, the proposed method does not require new additional redundancy bit sequence since the error correction applies the chaotic dynamics having the transmitted signal blocks.

As results of the computer simulation, we have confirmed that the advantage gained in BER performance of the proposed error-correcting method is about 1–1.5 dB compared to conventional method. Namely, we have achieved that the error-correcting scheme is performed without new additional redundancy bit sequence by using the chaotic dynamics. However, we have also found that the effect of the proposed scheme decreases according to increase in N. The reason for this is considered that central samples of each sequence influence the error correction when N becomes large. Therefore, to design the error-correcting scheme with central samples of chaotic sequences is our future problem. Moreover, we consider that analyzing the scheme's capability, such as the computational cost, is also important future work.

References

[1] M.J. Ororzalek, *Chaos and Complexity in Nonlinear Electronic Circuits*, World Scientific Publishing Co. Pte. Ltd., 1997.

[2] T. Tél and M. Gruiz, *Chaotic Dynamics*, Cambridge University Press, 2006.

[3] G. Kolumbán, B. Vizvári, W. Schwarz, and A. Abel, "Differential chaos shift keying: A robust coding for chaos communication," *Proc. NDES'96*, pp. 87–92, June 1996.

[4] M. Hasler and T. Schimming, "Chaos communication over noisy channels," *Int. J. Bifurcation and Chaos*, vol. 10, no. 4, pp. 719–736, April 2000.

[5] M. Hasler and T. Schimming, "Optimal and suboptimal chaos receivers," *Proc. IEEE*, vol. 90, no. 5, pp. 733–746, May 2002.

[6] F.C.M. Lau and C.K. Tse, *Chaos-Based Digital Communication Systems*, Springer, 2003.

[7] F.C.M. Lau and C.K. Tse, "Approximate-optimal detector for chaos communication systems," *Int. Journal of Bifurcation and Chaos*, vol. 13, no. 5, pp. 1329–1335, May 2003.

[8] F.C.M. Lau and C.K. Tse, "On optimal detection of noncoherent chaos-shift-keying signals in a noisy environment," *Int. J. Bifurcation and Chaos*, vol. 13, pp. 1587–1597, 2003.

[9] W.M. Tam, F.C.M. Lau, and C.K. Tse, "Generalized correlation-delay-shift-keying scheme for noncoherent chaos-based communication systems," *IEEE Trans. Circuits and Systems Part I*, vol. 53, no. 3, pp. 712–721, March 2006.

[10] L.E. Larson, J-M. Liu, and L.S. Tsimring, *Digital Communications Using Chaos and Nonlinear Dynamics*, Springer, 2006.

[11] P. Stavroulakis, *Chaos Applications in Telecommunications*, Talor & Francis Group, 2006.

[12] J.C. Fang and C.K. Tse, *Reconstruction of Chaotic Signals with Applications to Chaos-Based Communications*, Tsinghua University Press and World Scientific Publishing Co. Pte. Ltd., 2008.

[13] S. Arai and Y. Nishio, "Suboptimal receiver using shortest distance approximation method for chaos shift keying," *RISP J. Signal Processing*, vol. 13, no. 2, pp. 161–169, March 2009.

[14] S. Arai and Y. Nishio, "Noncoherent correlation-based communication systems choosing different chaotic maps," *Proc. ISCAS'07*, pp. 1433–1436, May 2007.

[15] S. Arai, Y. Nishio, T. Yamazato, and S. Ozawa, "M-ary modulation scheme using separation of chaotic dynamics for noncoherent chaos-based communications," *Proc. NOLTA'09*, pp. 312–315, October 2009.