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Preliminary Study on BPSK Receiver using Stochastic Resonance

Hiroya Tanaka†, Takaya Yamazato†† and Shintaro Arai†††

†School of Engineering
Nagoya University
C3-1(631) Furo, Chikusa, Nagoya, Japan

††Institute for Liberal and Sciences
Nagoya University
C3-1(631) Furo, Chikusa, Nagoya, Japan

†††Kagawa National College of Technology
551 Kohda, Takuma, Mitoyo, Japan

Abstract

Stochastic Resonance (SR), known as a noise-enhanced phenomenon, can improve the performance of communication systems. In this paper, we present a preliminary study on SR and its application to BPSK receiver. We discuss a basic question arises from a receiver using SR that shall we perform SR process in a radio-frequency (RF) band or in a baseband (BB)? As results, we have found that SR process in RF band shows better Bit Error Rate (BER) performance. This comes from the fact that a down conversion process that reduces the signal amplitude causes weak noise suppression by SR. While a double frequency term obtained also by the down conversion process do not affect much to the noise reduction performance.

1. Introduction

The concept of stochastic resonance (SR) was originally focused on the way of explaining the periodicity of primary cycle of recurrent ice ages in 1982. SR provided a simple, although not conclusive answer to this phenomenon [1, 2, 3]. In 1993, SR was demonstrated in mechanoreceptor neurons located in the tail fan of crayfish, and it has been taken its application to signal processing [4, 5].

The essential ingredients for SR consist of bistable system, with two inputs and with an output that is some function of the inputs and the internal dynamics of the system. For two inputs, a coherent signal and random noise are usually considered. As we understand, for a system well characterized by linear-response theory, an increase in input noise will results in a decrease in the output signal-to-noise ratio (SNR). In contrast, because of non-linear nature of SR, an increase in input noise can improve the output SNR [1, 2, 3]. To the authors' knowledge, this amazing feature of SR, the noise suppression, has been disregarded many researchers in the filed of communication engineering [5].

As a nature of communication systems, this noise input can be the background noise and an application of SR to communication system is straightforward. Another aspect of communication systems using SR is that some non-linear circuits used for communication systems, such as a comparator and a hard-limiter, exhibit SR phenomenon.

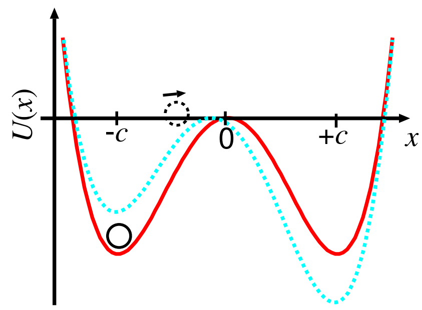

Figure 1: Double well potential minima at $\pm c$.

In this paper, as a preliminary study on an application of SR to communication systems, we present a bi-polar pulse receiver and BPSK receiver using SR. We discuss two receivers that perform SR process in a radio-frequency (RF) band and in a baseband (BB).

The paper is organized as follows: In Sec. 2, we briefly explain SR modeled by a classical double-well potential [1]. In Sec. 3, we present a bi-polar pulse receiver and how SR can improve the output SNR by an increase of input noise variance. In Sec. 4, we present a BPSK transmitter/receiver using SR. Here we compare BPSK receiver with SR process in a RF band and that in a BB. In Sec. 5, we evaluate BER performance of two receivers in Sec. 4 and discuss how their received signals affect their performances. Finally, Sec. 6 concludes the paper.

2. Stochastic Resonance

We consider the stochastic process modeled as a double-well form, as shown in Fig. 1. The process can be written by

$$\frac{dx}{dt} = -\frac{\partial U(x)}{\partial x} + s(t) + n(t), \quad (1)$$

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4, \quad (2)$$

where x is the output of SR, $s(t)$ is the input of SR, $n(t)$ is zero-mean white Gaussian noise with variance σ^2, and $U(x)$ is a potential of SR. This bistable double well potential is well used for an explanation of SR mechanism.

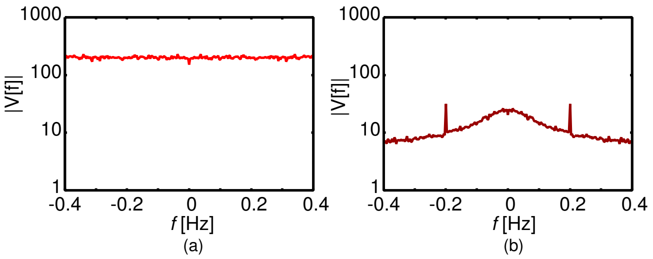

Figure 2: Frequency spectrum of SR: (a) Input, (b) Output.

In Fig. 1, the x-axis represents the SR output and y-axis represents its potential $U(x)$. If no noise and no periodic forcing, a particle spend most of time near $\pm c$. As the input noise variance increases, an occasional transition over the barrier in the center occurs. The rate at which such jumps occur will increase with the input noise variance[1].

Now if we add a periodic signal with small signal amplitude, then the potential will tilt first to the right in a first cycle and then to left in a next cycle. The periodic signal has the effect of modulating the transition rate.

2.1. Noise suppression by SR

Let small amplitude sinusoid signal of Eq. 1 be

$$s(t) = 0.4\cos(0.4\pi t), \tag{3}$$

and $\sigma^2 = 1$, $a = 1$ and $b = 1$. Figure 2 shows the spectrum of the SR input and the SR output. In this case, the input signal is completely buried in noise as shown in Fig. 2(a), the spectrum of the SR input (signal plus noise) is nothing but noise. However, if we observe the SR output as shown in Fig. 2(b), we see peaks near $f = \pm 0.2$Hz. We therefore confirm the effect of SR that it suppresses the input noise.

3. Bi-polar pulse transmitter/receiver

We present a bi-polar pulse transmitter/receiver using SR as shown in Fig. 3. For the receiver, the pulse signal with the amplitude A buried in an additive noise is passed to the SR process.

Let $b(t)$ be the input data signal given by

$$b(t) = \sum_i d_i \psi(t - iT_b), \tag{4}$$

where $d_i \in \{+A, -A\}$ is the data sequences, i is the index of time, T_b is the duration time of the data, and $\psi(s)$ is a bi-polar pulse $\psi(s) = 1$ in $0 \leq s < T_b$ and $\psi(s) = 0$ otherwise.

At the channel, the background noise $n(t)$ assumed to be zero-mean white Gaussian noise with variance σ^2 is added and then the received signal composed of $b(t)$ and $n(t)$ is fed into the SR process to suppress the background noise.

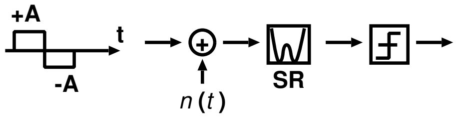

Figure 3: Bi-polar pulse receiver using SR.

3.1. The SNR improvement

The SNR curves at the SR output are shown in Fig. 4. We denote the SNR from the power spectrum density (PSD) of the SR output that is given by integrating Eq. 1. We set the amplitude of the input signal $A = 8/\sqrt{2}$, the sampling interval is 0.005sec, and the potential parameters $a = 32$, $b = 1$ in Eq. 2.

In Fig. 4, we denote the result given by a simulation with points and the result given by the theory with solid line. The theoretical SNR is expressed by rewriting (5.9) in [1]

$$\mathrm{SNR} = \frac{\sqrt{2}a^2A^2}{b\sigma^4}\mathrm{e}^{-\frac{a^2}{2b\sigma^2}}, \tag{5}$$

and by considering the signal bandwidth, the SNR is obtained by (4.4) in [1],

$$\mathrm{SNR'} = \frac{0.67}{2\pi\Delta}\mathrm{SNR} + 1, \tag{6}$$

where Δ is the bandwidth of the input signal. Because the input signal has both the signal component and the noise component, we add " +1 " to SNR. Equation 5 is given by computing the PSD of the SR output in adiabatic theory with Kramers rate in a double-well potential [1].

As we see from Fig. 4, the SNR curve is improved by an increase of noise variance σ^2. In this case the SNR at the SR input ($\mathrm{SNR_{in}}$) is about $-14.9 \sim -11.9$dB around the maximum for E_b/N_0 at the SR output. Here E_b is the bit energy, $N_0/2 = \sigma^2$ and $\mathrm{SNR_{in}}$ is shown as by rewriting (48) in [4]

$$\mathrm{SNR_{in}} = \frac{(A^2/2)}{(\sigma^2/dt)\Delta}, \tag{7}$$

where dt is the sampling interval. We, therefore, find the SR input is buried in noise and the range of E_b/N_0 that suppress the noise in the very narrow range, $-14.9 \sim -11.9$dB.

3.2. BER performance

We show the BER performance in Fig. 5. The simulation result is presented in dashed line, and the theoretical result is presented in solid line. The theoretical result is given by substituting the SNR of Eq. 6 to the equation BER = $0.5\mathrm{erfc}\sqrt{\mathrm{SNR}}$. In Fig. 5 the BER performance is improved by an increase of noise variance.

4. BPSK transmitter/receiver

A basic question arises from a receiver using SR is that shall we perform SR process in a radio-frequency (RF) band or in a baseband (BB)?

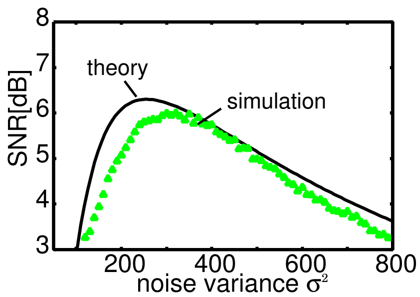

Figure 4: SNR for SR characteristic.

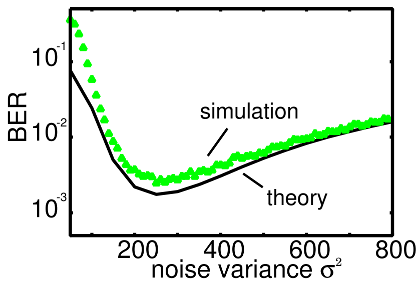

Figure 5: BER performance of bi-polar pulse receiver using SR.

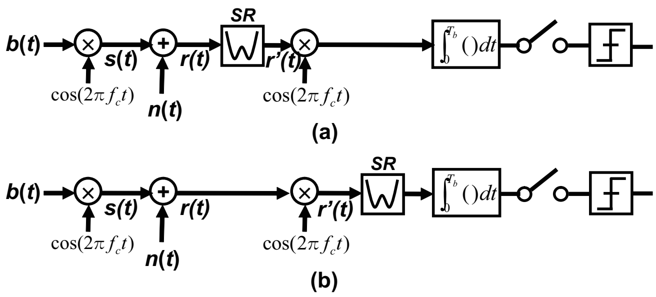

Figure 6: BPSK receiver model using SR: (a) SR is used before the carrier being multiplied, (b) SR is used after the carrier being multiplied.

In this section, we present a simple BPSK transmitter and two BPSK receivers using SR, as shown in Fig. 6. We consider two situations:
(1) The received signal is first passed to the SR process prior to a down conversion process as shown in Fig. 6(a), i.e., the noise suppression by the SR process is performed in RF band.
(2) The received signal is first down-converted to BB and performs the noise suppression by the SR process as shown in Fig. 6(b), i.e., the noise suppression process is performed in BB.

In this section, we focus on those two SR receivers and discuss their performance in terms of BER.

4.1. BPSK Transmitter

We consider the classical problem of detecting the presence of a known deterministic signal $s(t)$ buried in an additive input noise $n(t)$.

Using the input data signal $b(t)$, given by Eq. 4, and the carrier f_c, the transmitted signal $s(t)$ is given by

$$s(t) = b(t)\cos(2\pi f_c t). \tag{8}$$

At the channel, the background noise is added and some phase-shift is given. The received signal is given by

$$r(t) = b(t)\cos(2\pi f_c t + \theta) + n(t), \tag{9}$$

where $n(t)$ is zero-mean white Gaussian noise with variance σ^2 and θ is a phase-shift.

4.2. BPSK receiver using SR in RF band

For the receiver shown in Fig. 6(a), the received signal $r(t)$ is first passed to the SR process prior to multiplication of the carrier, i.e., the noise suppression is performed in RF region.

If we denote $r'(t)$ as the signal passed through the SR process, then $r'(t)$ can be calculated by Eq. 1 and Eq. 2. We then multiply the carrier, $\cos(2\pi f_c t + \theta)$, to down convert the signal.

$$\begin{aligned} & r'(t)\cos(2\pi f_c t + \theta) + n'(t) = \\ & \frac{b(t)}{2} + \frac{b(t)}{2}\cos 2(2\pi f_c t + \theta)) + n'(t) \end{aligned} \tag{10}$$

Here $n'(t)$ represents the down-converted and the SR processed noise term $n(t)$. Note that we assume the SR process does not affect the signal component, but only reduces the given noise variance.

Finally, the data is recovered by the integrated and dump circuit and a threshold decision.

In this case, the noise suppression is performed in RF region and the received signal may be treated as it is. The demodulation process is performed after the SR process. From a hardware perspective, it may be difficult to implement a SR process in RF region.

4.3. BPSK receiver using SR in BB

For the receiver shown in Fig. 6(b), the received signal is first down-converted to BB and performs the noise suppression process by the SR process, i.e., the noise suppression is performed in BB. In this case, we simply replace $r'(t)$ and $n'(t)$ in Eq. 10 to $r(t)$ and $n(t)$. Focusing only on the signal terms, we have sum of $b(t)/2$ (the amplitude is a half) and $\frac{b(t)}{2}\cos 2(2\pi f_c t + \theta)$ (the double frequency term). These two are fed into the SR process for the noise suppression.

5. BER evaluation by simulation

In this section we evaluate BER performance of two receivers shown in Fig. 6. We set the amplitude of the input

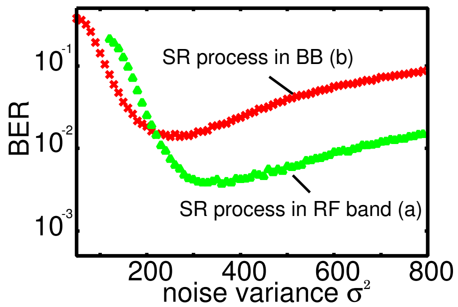

Figure 7: Simulation result for BER of BPSK receiver using SR in Fig.6

data $A = 8$, the data duration $T_b = 160$sec, the carrier frequency $f_c = 0.2$Hz, the sampling interval is 0.005sec, and the potential parameters $a = 32$, $b = 1$ in Eq. 2.

Figure 7 shows the BER versus the noise variance σ^2. As we see from Fig. 7, the both BER curves are improved by an increase in σ^2, and then they slightly degrade depending on a further increase in σ^2. For the receiver having the SR process in RF band (Fig. 7(a)), it achieves its minimum BER at $\sigma^2 \simeq 300$, while for the receiver having the SR process in BB (Fig. 7(b)), it achieves its minimum BER at $\sigma^2 \simeq 250$. We also confirm that for the receiver having the SR process in RF band, the BER curve slightly shift to right of the BER curve of the receiver with the SR process in BB and it is better than the receiver with the SR process in BB.

This performance difference may come from where we perform the noise suppression by the SR, i.e., the input to the SR having sum of $b(t)/2$ (the half amplitude term) and $\frac{b(t)}{2}\cos 2(2\pi f_c t + \theta)$ (the double frequency term) or not.

Here, we evaluate how the signal amplitude and frequency would affect the noise suppression process of the SR. Figure 8(a) shows the BER performance in Fig. 6(a) for the SR input signal having $f_c = 0.2$Hz and $f_c = 0.02$Hz. As we see from the figure, two BER curves show same tendency that they improves as an increase of σ^2. If we compare those two curves, we see that the curves with $f_c = 0.2$Hz slightly shift to right and the BER is worse than the case with $f_c = 0.02$Hz. This agrees with the results discussed in [1] that the SR can be modeled as LPF. However, the result disagrees with the BER shown in Fig. 7 that the SR process in RF band shows better BER performance.

Next, we evaluate the effect of amplitude. Figure 8(b) shows the BER performance in Fig. 6(a) for the SR input signal having $A = 8$ and $A = 10$. Note that the SNRs observed prior to the SR process are -13.7dB and -11.8dB at $\sigma^2 = 300$ given by Eq. 7. Therefore, both input signals are buried in noise. From Fig. 8(b), we see the BER improvement by the increase of the amplitude from 8 to 10. We therefore confirm that more effect of the signal amplitude than the signal frequency, and the term $\frac{b(t)}{2}$ in Eq. 10 has large effect on

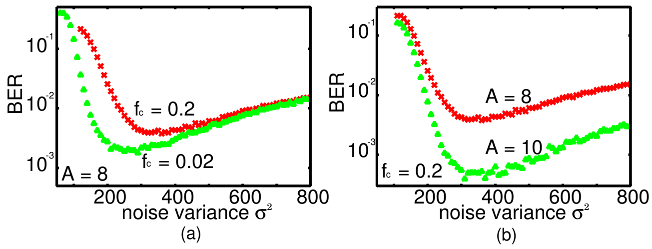

Figure 8: Comparison for BER by varying the input amplitude and carrier frequency.

the BER performance.

6. Conclusions

In this paper, we have investigated SR application to bi-polar pulse receiver and BPSK receiver. For bi-polar pulse receiver, we have found that the SR can improve the output SNR and BER performance as well by an increase of input noise variance. For BPSK receiver, we have also confirmed the noise suppression by the SR. As results, the receiver using SR in RF band shows better BER performance than the receiver using SR in BB. This comes from the fact that a down conversion process that reduces the signal amplitude causes weak noise suppression by SR. While a double frequency term obtained also by the down conversion process do not affect much to the noise reduction performance.

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References

[1] Bruce McNamara and Kurt Wiesenfeld, "Theory of stochastic resonance," PHYSICAL REVIEW A, Vol. 39, No. 9, pp. 4854-4869, May. 1989.

[2] Luca Gammaitoni, Peter Hänggi, Peter Jung, and Fabio Marchesoni, "Stochastic resonance," Rev. Mod. Phys., Vol. 70, No. 1, pp. 223-287, Jan. 1998.

[3] Adi R.Bulsara and Luca Gammaitoni, "TUNING INTO NOISE," PHISICS TODAY, pp 39-45, Mar. 1996.

[4] François Chapeau-Blondeau and Xavier Godivier, "Theory of stochastic resonance in signal transmission by static nonlinear systems," PHYSICAL REVIEW E, Vol. 55, No. 2, pp. 1478-1495, Feb. 1997.

[5] Yukihiro TADOKORO, Akihisa ICHIKI and Masaaki TAKAHASHI, "A Sampling Frequency and Observation Time Analysis for Efficient Stochastic Resonance in Signal Processing," Proc. of ICNF'11, pp. 176-179, Jun. 2011.