# Accuracy Improvement by Phase Only Correlation for Distance Estimation Scheme for Visible Light Communications Using an LED Array and a High-speed Camera 

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#### Abstract

In this paper, we focus on a distance estimation scheme using camera. Our distance estimation scheme is based on triangulation. The accuracy of camera based scheme is governed by resolution of a camera, or equivalently the size of pixel. Therefore we introduce phase only correlation (POC) to estimate with subpixel accuracy. As a result of experiment, we will show that our scheme achieves less than 0.3 m estimation error at the distance of 60 m . By using POC, the resolution of estimation reaches nearly 0.02 pixel, much less than one pixel.


Keywords: Visible Light Communication, Distance Estimation, High-speed camera, Phase-Only Correlation

## 1. INTRODUCTION

Along with the popularization of LED devices, visible light communications (VLC) using LED have attracted a great deal of attention as novel communication systems. Especially, VLC applications using LED traffic lights are one of interesting topics in the field of intelligent transport systems (ITS) [1]. We focus our attention on utilizing distance estimation

Distance estimation from a traffic light to a vehicle is one of important techniques in ITS. This technique is essential to realize stop position assistance and correct guidance of navigation system. Accordingly, a number of techniques to estimate distance have been developed, such as GPS [2], six axis sensor [3], using photo diode and LED head lamp [1]. Unfortunately, the accuracy of GPS may not be enough and photo diode cannot be used under sun. Camera is also popular device for distance estimation. As most of distance estimation scheme using camera is based on triangulation, those methods use at least one object and two cameras or two objects and one camera [4] [5]. Therefore a challenge is how to estimate distance with one object and one camera. We further note that the accuracy of camera based scheme is governed by resolution of camera, or equivalently the size of pixel.

In this paper, we propose a distance estimation scheme using one object and one camera. In particular, we consider an LED traffic light as the transmitter, and an in-vehicle high-speed camera as the receiver. Further, we introduce phase only correlation (POC) to estimate with subpixel accuracy. Subpixel is unit which is smaller than pixel. As a result of experiment, we will show that our scheme achieves less than 0.3 m estimation error at the distance of 60 m . By using POC, the resolution of estimation reaches nearly 0.02 pixel, much less than one pixel.

The paper is organized as follows. In Sec. 2, we introduce system model of visible light communication and distance estimation system. In Sec. 3, we explain the distance estimation scheme, utilizing phase only correlation and the method using phase only correlation to estimate width of LED array with subpixel accuracy.
Section 4 shows experimental result. Finally, we conclude this paper in Sec. 5.

## 2. SYSTEM MODEL

In this section, we first explain visible light communication system that we consider in this paper [6]. We utilize a distance estimation scheme in VLC. We consider an LED traffic light (LED array)as the transmitter, and an in-vehicle high-speed camera as the receiver. Figure 1 shows the block diagram of the system.

The transmitter modulates each of LEDs independently. In other words, the transmitter generates a two-dimensional (2D) LED pattern according to input data. For distance estimation purpose, we insert a particular 2D LED pattern as a part of data input. Further, we add width of the LED array data required for the distance estimation.

In the receiver, the transmitted 2D LED pattern passes through the optical channel and it is captured by the in-vehicle high-speed camera. The distance estimation unit determines the
position of each LED and extracts the LED luminance. Using the position of LED, its luminance of the transmitted 2D LED pattern, and the width of the LED array, we calculate distance.


Figure 1. Block diagram of distance estimation scheme.

## 3. DISTANCE ESTIMATION

### 3.1 Distance estimation scheme using pixel-width of LED-array

In this paper, we propose the distance estimation scheme based on triangulation. Normally, when we use triangulation, either two transmitters and one camera, or one transmitter and two cameras are needed. In this paper we propose to use one LED array and one camera. In particular, we use pixel-width of LED array as shown in Fig.2. Here we define the pixel-width as a number of pixels representing a width or length of a target LED array. For simplicity, let us represents $W$ as the width of the LED array in the horizontal direction of the image.


Figure 2. Pixel-width $W$ and 2D LED pattern $f_{1}(m, n)$ and $f_{2}(m, n)$, transmitted for the distance estimation purpose.


Figure 3. Distance estimation scheme based on using triangulation.

Figure 3 shows an illustrative drawing of the distance estimation. The distance estimation process of the proposed scheme is as follows:

1) At the transmitter we blink two faraway LEDs one-by-one. Two images which captures these LED are $f_{1}(m, n)$ and $f_{2}(m, n)$. Figure 2 shows $f_{1}(m, n)$ and $f_{2}(m, n)$ and resolution of the images is $P \times P$.
2) We measure pixel-width $W$ of two LEDs. Since the luminance value of LED turning on is high compared to the other background, measuring $W$ can be processed by counting the number of pixels of the highest luminance pixel of two captured images.
3) Finally we obtain $L$ by calculating as follows,

$$
\begin{equation*}
L=\frac{S P}{2 W \tan \frac{\theta}{2}}=\frac{f S}{W \alpha} \tag{1}
\end{equation*}
$$

where $S$ is the width of the LED array, $\alpha$ is a pixel size of the camera, $f$ is the focal distance of the camera lens. The angle of view $\theta$ is

$$
\begin{equation*}
\tan \frac{\theta}{2}=\frac{\alpha P}{2 f} \tag{2}
\end{equation*}
$$

If we obtain $W$ then we can estimate $L$. In this method, because $S, \alpha, f$ are constants, the accuracy of distance is governed by $W$.

Figure 4 illustrates $3 \times 3$ pixels around the lightning LED that is shown by white circle in Fig. 2. The LED overlaps with six pixels. We estimate position of LED as the highest luminance pixel in six pixels. Then, the estimated center of LED is considered as the actual center of pixel shown in circle; on the other hand, the center of LED is shown by a triangle. The actual center of LED is different from the estimated center of the pixel. Because error in estimating the actual center of LED is at most 0.5 pixel and we need two LED images to derive $W$, the error is at most one pixel.

Now let us calculate how this error would affect to the distance estimation. Figure 5 shows the distance estimation error versus distance for the case of 1 pixel error, 0.5 pixel error and 0.1 pixel error. We calculate distance estimation error $\hat{L}$ when each pixel error occurs. Distance estimation error $L^{\prime}$ is given by calculating Eq. (1),

$$
\begin{equation*}
L^{\prime}=|\hat{L}-L|=\left|\frac{f S}{\left(W+W_{e}\right) \alpha}-L\right| \tag{3}
\end{equation*}
$$

where $W_{e}\left(0<W_{e}<1\right.$ pixel $)$ is difference between the center of LED and estimated center of LED, and we use the parameters of the experimental instruments $f, S$ and $\alpha$ listed in Table 1. We note that these performance curves are the case of worst estimation.

Here, we focus attention on stop position assistance in a crossing. When we stop in a crossing, it is necessary to stop $1.0 \mathrm{~m}-1.5 \mathrm{~m}$ away from a stop line or a car. In addition, the distance from traffic lights to stop line of the largest crossing (7 traffic lanes in one side) in Japan is approximately 60 m . From these two conditions, it is necessary to estimate distance estimation error less than 0.5 m at 60 m . In Fig.5, the performance curve satisfied with this condition is the curve of 0.1 pixel error. Therefore we need $W_{e} \leqq 0.1$ pixel.


Figure 4. Captured LED overlap with multiple pixels.


Figure 5. Performance curves are the case of worst estimation calculated by Eq. (1).

### 3.2 Phase only correlation

We use the phase only correlation (POC) to estimate the pixel-width of LED array [7]-[9]. POC is a pattern matching algorithm and defined as modified correlation algorithm such that the amplitude components of the Fourier transformed images are replaced with a constant. Generally, POC is used for the position displacement detection of the same object. We show processing algorithm in Fig. 6.


Figure 6. Phase Only Correlation.
Let two captured LED images consisting of $M \times N$ pixels be $f_{1}(m, n)$ and $f_{2}(m, n)$. Those images are ones transmitted for the distance estimation purpose shown in Fig.3. Two images are natural images and index ranges are $m=0,1,2, \cdots, M-1$ and $n=0,1,2, \cdots, N-1$. Let the discrete Fourier transform image of $f_{1}(m, n), f_{2}(m, n)$ be $F_{1}(u, v), F_{2}(u, v)$. we obtain them as follows;

$$
\begin{align*}
& F_{1}(u, v)=\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_{1}(m, n) e^{-j 2 \pi\left(\frac{m u}{M}+\frac{n v}{N}\right)}=A(u, v) e^{j \theta(u, v)}  \tag{4}\\
& F_{2}(u, v)=\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_{2}(m, n) e^{-j 2 \pi\left(\frac{m u}{M}+\frac{n v}{N}\right)}=B(u, v) e^{j \varphi(u, v)} \tag{5}
\end{align*}
$$

where $u=0,1,2, \cdots, M-1, v=0,1,2, \cdots, N-1$, and $A(u, v), B(u, v)$ are amplitude components, and $\theta(u, v), \varphi(u, v)$ are phase components.

We replace the amplitude components $A(u, v), B(u, v)$ of $F_{l}(u, v), F_{2}(u, v)$ with one. We acquire an image (phase image) with only phase information by replacing the amplitude
components with one. Phase images $F^{\prime}{ }_{1}(u, v)$ and $F^{\prime}{ }_{2}(u, v)$ corresponding to $F_{1}(u, v)$ and $F_{2}(u$, $v$ ) are given by

$$
\begin{align*}
& F_{1}^{\prime}(u, v)=e^{j \theta(u, v)},  \tag{6}\\
& F_{2}^{\prime}(u, v)=e^{j \varphi(u, v)} . \tag{7}
\end{align*}
$$

We get a synthetic image $H_{12}(u, v)$ by multiplying complex conjugate of phase image $F^{\prime}{ }_{1}(u, v), F^{\prime}{ }_{2}(u, v)$. It is given by

$$
\begin{equation*}
H_{12}(u, v)=F_{1}^{\prime}(u, v)\left(F_{2}^{\prime}(u, v)\right)^{*}=e^{j(\theta-\varphi)} \tag{8}
\end{equation*}
$$

Phase only correlation $G_{12}(r, s)$ is calculated by inverse Fourier transform using synthetic image $H_{12}(u, v)$,

$$
\begin{align*}
G_{12}(r, s) & =\sum_{u=0}^{M-1} \sum_{v=0}^{N-1}\left(H_{12}(u, v)\right) e^{-j 2 \pi\left(\frac{u r}{M}+\frac{v s}{N}\right)}  \tag{9}\\
& =\sum_{u=0}^{M-1} \sum_{v=0}^{N-1}\left(e^{j(\theta-\varphi)}\right) e^{-j 2 \pi\left(\frac{u r}{M}+\frac{v s}{N}\right)}
\end{align*}
$$

where $r=0,1,2, \cdots, M-1$ and $s=0,1,2, \cdots, N-1$.

By setting $f_{2}(m, n)=f_{1}(m, n)$, we obtain autocorrelation

$$
G_{11}(r, s)= \begin{cases}1 & r=0, s=0  \tag{10}\\ 0 & r \neq 0, s \neq 0\end{cases}
$$

An example of result is shown in Fig. 7.


Figure 7. Autocorrelation

$$
f_{2}(m, n)=f_{1}(m, n)
$$



Figure 8. POC image $\boldsymbol{G}_{12}$ for $f_{1}(m, n)$ and $f_{2}(m, n)=f_{1}\left(m+\tau_{m}, n+\tau_{n}\right)$.

In POC, the position displacement of the two images is reflected by cross-correlation. Let us consider $f_{2}(m, n)=f_{1}\left(m+\tau_{m}, n+\tau_{n}\right)$ that the original image $f_{1}(m, n)$ is shifted to $\tau_{m}$ in $m$ direction and $\tau_{n}$ in $n$ direction, respectively where $\tau_{m}$ and $\tau_{n}$ are consecutive values. Note that, pixel-width $W$ takes a natural number. If we denote $\delta_{m}$ and $\delta_{n}$ as a small discrete value, $\tau_{m}$ and $\tau_{n}$ are given by

$$
\begin{align*}
\tau_{m} & \cong W_{m}+\delta_{m} \\
\tau_{n} & \cong W_{n}+\delta_{n} \tag{11}
\end{align*}
$$

where equality of Eq.(11) is obtained by infinitely small $\delta$. By using $\tau_{m}$ and $\tau_{n}$, we can rewrite

Eq. (5) as follows.

$$
\begin{align*}
F_{2}(u, v) & =\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f_{2}\left(m+\tau_{m}, n+\tau_{n}\right) e^{-j 2 \pi\left(\frac{\left(m+\tau_{m}\right) u}{M}+\frac{\left(n+\tau_{n}\right) v}{N}\right)}  \tag{12}\\
& =B(u, v) e^{j\left(\varphi+\frac{2 \pi \tau_{m} u}{M}+\frac{2 \pi \tau_{n} u}{N}\right)}
\end{align*}
$$

Then, phase image $F^{\prime}{ }_{2}(u, v)$ of $f_{2}(m, n)$ becomes

$$
\begin{equation*}
F_{2}^{\prime}(u, v)=e^{j\left(\varphi+\frac{2 \pi \tau_{m} u}{M}+\frac{2 \pi r_{n} u}{N}\right)}, \tag{13}
\end{equation*}
$$

and synthetic image $H_{12}(u, v)$ becomes

$$
\begin{align*}
H_{12}(u, v) & =F_{1}^{\prime}(u, v)\left(F_{2}^{\prime}(u, v)\right)^{*} \\
& =e^{j\left(\theta-\phi-\frac{2 \pi \tau_{m} u}{M}-\frac{2 \pi t_{n} u}{N}\right)}, \tag{14}
\end{align*}
$$

and phase only correlation $G_{12}(r, s)$ becomes

$$
\begin{align*}
G_{12}(r, s) & =\sum_{u=0}^{M-1} \sum_{v=0}^{N-1}\left(H_{13}(u, v)\right) e^{j 2 \pi\left(\frac{u r}{M}+\frac{v s}{N}\right)} \\
& =\sum_{u=0}^{M-1} \sum_{v=0}^{N-1}\left(e^{j\left(\theta-\varphi-\frac{2 \pi \tau_{m} u}{M}-\frac{2 \pi \tau_{n} u}{N}\right)}\right) e^{j 2 \pi\left(\frac{u r}{M}+\frac{v s}{N}\right)}  \tag{15}\\
& =G_{11}\left(r-\tau_{m}, s-\tau_{n}\right) \cong \begin{cases}1 & r=W_{m}+\delta_{m}, s=W_{n}+\delta_{n} \\
\xi(<1) & r=W_{m}, s=W_{n} \\
\varepsilon & r \neq W_{m}, s \neq W_{n}\end{cases}
\end{align*}
$$

where $\xi$ is a constant and we assume $\xi \gg \varepsilon$, for a small $\varepsilon$.

Figure. 8 shows a result of the POC using two images. Unfortunately, from Eq. (15) $r$ and $s$ take only integer values of $W_{m}$ and $W_{n}$, respectively, so the accuracy is still limited to the pixel-width. Since the actual values of displacement are denoted by $\tau_{m} \cong W_{m}+\delta_{m}$ and $\tau_{n} \cong W_{n}+\delta_{n}$, as shown in Eq. (11), it is necessary to estimate $\delta_{m}$ and $\delta_{n}$.

### 3.3 Estimation of $\boldsymbol{\delta}_{\boldsymbol{m}}$ and $\boldsymbol{\delta}_{\boldsymbol{n}}$

From Eq. (15), if we know $\delta_{m}$ and $\delta_{n}$ then $G_{12}(r, s)$ takes one. Farther, we can approximate $G_{12}(r, s)$ by sinc function.

$$
\begin{equation*}
G_{12}(r, s) \cong \operatorname{sinc}\left(r+\delta_{m}\right) \operatorname{sinc}\left(s+\delta_{n}\right) \tag{16}
\end{equation*}
$$

The peak of $G_{12}(r, s)$ can be obtained by varying $\delta_{m}, \delta_{n}$. The center of LED is given by,

$$
\begin{equation*}
\left(W_{\mathrm{m}}+\hat{\delta}_{m}, W_{\mathrm{n}}+\hat{\delta}_{n}\right)=\underset{-1 \leq \delta_{m} \leq 1,-1 \leq \delta_{n} \leq 1}{\arg \max } G_{12}(r, s) \tag{17}
\end{equation*}
$$

where $\hat{\delta}_{m}$ and $\hat{\delta}_{n}$ are the estimates of $\delta_{m}$ and $\delta_{n}$, respectively. Figure 10 shows cross-section of Fig. 9 in $n$ direction. The triangle in Fig. 9 expresses center of LED and $W_{\mathrm{n}}+\hat{\delta}_{n}$ for Eq. (17)
in $n$ direction.
In Eq. (11), estimation accuracy improves by estimating $\delta_{m}$ and $\delta_{n}$.


Figure 9. Sinc function approximation of $G_{12}(r, s)$ shown in cross-section of Fig. 9 in $\boldsymbol{n}$ direction

## 4. EXPERIMENT

We performed measurements for distances from 20 m to 60 m , every 5 m on a static condition. We evaluate the results of experiments by distance estimation error. Figure 10 (a) and (b) show experimental instruments; the LED array transmitter, the in-vehicle high-speed camera connected to a PC. Table 1 (a) and (b) summarize the parameters of experimental instruments. As the in-vehicle high-speed camera, we use Photron FASTCAM-1024PCI 100K. We note that the used LEDs are the same as the actual LED traffic lights in Japan, but the LED spacing is less than 5 mm . For $\delta_{m}$ and $\delta_{n}$ of Eq. (17) we vary them with the step size of 0.1 pixel.

(a) LED array transmitter

(b) High-speed camera

Figure 10. Experimental instruments
Table 1. Parameters of the experimental instruments

| Number of LED | $32 \times 32$ |
| :--- | :--- |
| Spacing of LEDs | 15 mm |
| Size $(\mathrm{S} \times \mathrm{S})$ | $465 \mathrm{~mm} \times 465 \mathrm{~mm}$ |

(b) High-speed camera

| pixel size $\alpha$ | $17 \mu \mathrm{~m}$ |
| :--- | :--- |
| Focal distance $f$ | 35 mm |
| Captured | Monochrome |
| Lens diaphragm | 8 |
| Focus of a lens | Infinity |
| Resolution $(P \times P)$ | $128 \times 128$ pixel |



Figure 11. Average distance estimation error performance versus distance
Figure 11 illustrates the average distance estimation error performances. The horizontal axis expresses distance between the LED array and the high-speed camera. The vertical axis expresses the distance estimation errors. The graph label "pixel level estimation" represents the distance estimation method of Sec.3.1 using pixel-width. The graph label "Subpixel level estimation" represents the distance estimation error performance that of the distance estimation method of Sec.3.1-Sec.3.3. The curve of "Subpixel level estimation" is less than 0.3 m distance estimation error from 20 m to 60 m , while the curve of "Pixel level estimation" scheme shows more than 1 m distance estimation error at 60 m . As we see clearly from the figure, we can reduce the distance estimation error by estimating $\delta_{m}$ and $\delta_{n}$ in Eq. (17).

Along with those curves we plot the worst distance estimation performance calculated by Eq. (1) for the case of 0.3 pixel error and 0.02 pixel error. The error of "Subpixel level estimation" which is our proposed method almost coincides with the curve of " 0.02 pixel error". Hence the accuracy of our distance estimation method reaches nearly 0.02 pixel.

## 5. CONCLUSIONS

In this paper, we discussed distance estimation scheme using LED array and in-vehicle high-speed camera. The accuracy of camera based scheme is governed by resolution of camera, or equivalently the size of pixel. Further, we brought in POC to estimate with subpixel displacement ( $\delta_{m}$ and $\delta_{n}$ ). As a result of experiment, we showed that our scheme achieves less than 0.3 m estimation error at the distance of 60 m . The average distance estimation error performances almost coincides the worst distance estimation performance of 0.02 pixel error.

## 6. REFERENCES

[1]. Shogo Kitano, Shinichiro Haruyama, Masao Nakagawa, "Inter vehicle Communication and Ranging System using LED Headlamp" IEICE Technical Report ITS 2004-65, IN2004-199
[2]. Hiroyuki Uchiyama, Daisuke Deguchi, Tomokazu Takahashi, Ichiro Ide, and Hiroshi Murase "Vehicle Position Estimation using Street Image Map and Two In-vehicle Cameras", Recognition, Understanding symposium of the Image (MIRU2009), July, 2009
[3]. Mio Obata, Shinichiro Haruyama, Masao Nakagawa, "Highly Accurate Positioning Technique using Visible Light ID System and Six-Axis Sensor", IEICE Technical Report IN2006-254
[4]. Tsubasa Saito, Shinichiro Haruyama, Masao Nakagawa, "A Study for Inter-Vehicle Communication and Ranging Method using LED Rear Lights and Image Sensor", IEICE Technical Report, No.CAS2005-138, SIP2005-184, CS2005-131, pp.121-126, March, 2006.
[5]. Hideaki Uchiyama, Masaki Yoshino, Shinichiro Haruyama, Hideo Saito, Masao nakagawa, Takao Kakehashi, Naoki Nagamoto, " A Photogrammetric System Based on Visible Light Communications Using Light Markers"
[6]. Yasutake Shiraki et al. "Robust Receiver Design for Road-to-Vehicle Communication System Using LED Array and High-Speed Camera", 18th World Congress on Intelligent Transport Systems
[7]. Kenji Takita, Takafumi Aoki, et al,"High-Accuracy Subpixel Image Registration Based on Phase-Only Correlation", IEICE Transaction, Fundamentals, Vol.E86-A, No. 8 pp.1925-1933, August, 2003.
[8]. Koji Kobayashi, Hiroshi Nakajima, Takafumi Aoki, Masayuki Kawamata, Tatsuo Higushi, "Principles of Phase Only Correlation and Its Applications", ITE Technical Report, Vol.20, No.41, PP.16, MIP'96-53, NIM'96-75, July, 1996
[9]. Hassan Foroosh, Josaine B.Zerubia, Marc Berthod, "Extension of Phase Correlation to Subpixel Registration" IEEE Transactions on Image Processing, vol11, No3, March 2002

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