2007 IEEE Workshop on Nonlinear Circuit Networks
NCN'07, Tokushima, Japan, Dec 7--8, 2007

Improvement of Error Performance for Noncoherent Chaos Communications

Shintaro ARAI†, Yoshifumi NISHIO† and Takaya YAMAZATO‡
†Department of Electrical and Electronic Engineering, Tokushima University
2-1 Minami-Josanjima, Tokushima, 770-8506 JAPAN
{arai, nishio}@ee.tokushima-u.ac.jp
‡EcoTopia Science Institute, Nagoya University
Furo-cho, Chikusa-ku,Nagoya, 464-8603, JAPAN,
yamazato@nuee.nagoya-u.ac.jp

Abstract— **This paper proposes the error-correcting for noncoherent chaos communications. To improve the error performance, we focus attention on the successive chaotic sequence based on the chaotic dynamics. Concretely, error performance of a noncoherent receiver improves by analyzing the chaotic dynamics of the successive received sequence. As results of computer simulations, we confirm about 3 dB gain in BER performance compared to the conventional suboptimal receiver.**

I. Introduction

Recently, research on digital communications systems using chaos becomes a hot topic [1]− [3]. Especially, it is attracted to develop noncoherent detection systems which do not need to recover basis signals (unmodulated carries) at the receiver. In this study, we focus attention on the optimal receiver which is one of typical noncoherent systems [1]. The optimal receiver performs an optimal detection by using the probability density function (PDF) between the received signals and the same chaotic map of the transmitting side. However, the optimal receiver suffers from a computational complexity due to the large chaotic sequence length. Thus, it is important to develop a receiver with performance equivalent to the optimal receiver using different algorithms, i.e., a suboptimal receiver.

In our previous research, we proposed the suboptimal receiver using the shortest distance approximation [4]. Instead of calculating the PDF, the proposed suboptimal receiver approximates the PDFs by calculating the shortest distance between the received signals and the chaotic map. As results of the computer simulations, we confirmed the validity of the proposed suboptimal receiver as an approximation method of the optimal receiver.

In this study, to improve the error performance of the suboptimal receiver more, we propose an error-correcting method for the suboptimal receiver. Note that the detection characteristic of the suboptimal receiver is not superior to the optimal receiver. To solve this problem, we focus attention on the successive chaotic sequence based on the chaotic dynamics. Concretely, we can create the successive chaotic sequences having the same chaotic dynamics. This feature gives the receiver additional information to correctly recover the received noisy signal. Therefore, by analyzing the chaotic dynamics at the receiver, it is possible to improve the error performance. As results of computer simulations, we confirm about 3 dB gain in BER performance compared to the conventional suboptimal receiver.

II. System Overview

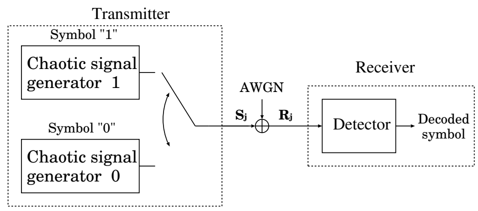


Fig. 1. Block diagram of discrete-time binary CSK communication system.

We consider the discrete-time binary CSK communication system, as shown in Fig. 1. In the transmitter, a chaotic sequence is generated by a chaotic map. In this study, we use a skew tent map which is one of simple chaotic maps, as described by Eq. (1)

$$x_{i+1} = \begin{cases} \dfrac{2x_i + 1 - a}{1+a} & (-1 \leq x_i \leq a) \\ \dfrac{-2x_i + 1 + a}{1-a} & (a < x_i \leq 1) \end{cases} \tag{1}$$

where a denotes a position of the top of the skew tent map. The information symbol is modulated by Chaos Shift Keying (CSK) which is a digital modulation system using chaos. When the transmitter generates the signals, we use chaotic sequences generated by different chaotic maps depending on the value of an information symbol. If the information symbol "1" is sent, Eq. (1) is used, and if "0" is sent, the reversed function of Eq. (1) is used. To transmit a 1-bit information, N chaotic signals are generated, where N is the chaotic sequence length.

In this study, to perform the error-correcting by the receiver, information are transmitted by a packet which consists of a data part and a redundancy part, as shown in Fig. 2, where $\mathbf{S_j}$ is the transmitted signal vector, $j = (0, 1, \cdots, K-1)$, K is total bit per packet, **Seq "1"** and **Seq "0"** are the chaotic sequence. Note that the transmitted signal vector $\mathbf{S_j}$ is different for each symbol.

[When the symbol "1" is sent]

$$\mathbf{S_j} = (x_j \ f^{(1)}(x_j) \cdots f^{(i)}(x_j) \cdots f^{(N-1)}(x_j)) \tag{2}$$

[When the symbol "0" is sent]

$$\mathbf{S_j} = (y_j \ g^{(1)}(y_j) \cdots g^{(i)}(y_j) \cdots g^{(N-1)}(y_j)) \tag{3}$$

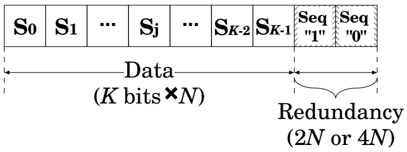


Fig. 2. Packet Format.

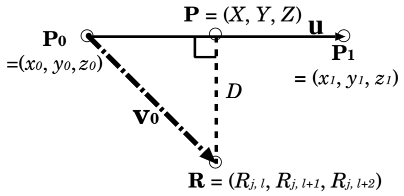


Fig. 3. Calculation of the shortest distance.

where $f^{(i)}$ and $g^{(i)}$ are the function of the skew tent map for symbol "1" and "0", i is the iteration of f or g, x_j or y_j denotes the initial value of the jth symbol = "1" or "0" respectively. When K bits data are transmitted, the length of the data part becomes $K \times N$. The redundancy is the sequence which has the known information symbol, and its length is different depending on the error-correcting method. In addition, the initial value of the chaotic sequence is also different in each chaotic signal generator and is updated each packet, namely, the chaotic sequence generated by each generator in the packet is a successive sequence based on the chaotic dynamics. As an example, let us assume the jth and $(j+3)$th symbols are "1" (i.e. the $(j+1)$th and $(j+2)$th symbols are "0"). In this case, x_{j+3} becomes $f^{(N)}(x_j)$. In the same way, y_{j+2} becomes $g^{(N)}(y_{j+1})$. Moreover, **Seq "1"** and **Seq "0"** are also a successive sequence based on the chaotic dynamics.

The channel distorts the signal and corrupts it by noise. In this study, noise of the channel is assumed to be the additive white Gaussian noise (AWGN). Thus, the received signals block is given by $\mathbf{R_j} = (R_{j,0}\ R_{j,1} \cdots R_{j,N-1}) = \mathbf{S_j} + AWGN$.

The receiver recovers the transmitted signals from the received signals and demodulates the information symbol. Also, the receiver performs the error-correcting in this study. Since we consider a noncoherent receiver, the receiver memorizes the chaotic map used for the modulation at the transmitter. However, the receiver never knows the initial value of chaos and the information symbol in the transmitter. Before explaining the error-correcting, we introduce the operation of our suboptimal receiver to be the basis for the proposed error-correcting.

Our suboptimal receiver calculates the shortest distance between the received signals and the chaotic map in the N_d-dimensional space using N_d successive received signals $(N_d : 2, 3, \cdots)$. As an example, we explain the case of $N_d = 3$. In this case, the receiver calculates the shortest distance between $\mathbf{R}$ and the chaotic map using the scalar product of the vector, where $\mathbf{R}$ is the three successive received signals $\mathbf{R} = (R_{j,l}, R_{j,l+1}, R_{j,l+2})$, $R_{j,l}$ denotes the jth symbol and lth signal of the received signals $(l = 0\ 1 \cdots N-3)$. Any two points of $\mathbf{P_0} = (x_0, y_0, z_0)$ and $\mathbf{P_1} = (x_1, y_1, z_1)$ are chosen from each straight line in the space, as shown in Fig. 3. Using Fig. 3, we can calculate $\mathbf{P} = (X, Y, Z)$ and the shortest distance D by the following equations.

$$\mathbf{P} = (X, Y, Z) = (\mathbf{u} \cdot \mathbf{v_0})\, \mathbf{u} + \mathbf{P_0} \tag{4}$$

$$D = ||\mathbf{P} - \mathbf{R}|| \tag{5}$$

where

$$\text{Unit vector } \mathbf{u} = \frac{\mathbf{P_1} - \mathbf{P_0}}{||\mathbf{P_1} - \mathbf{P_0}||} \tag{6}$$

$$\mathbf{v_0} = \mathbf{R} - \mathbf{P_0}\ . \tag{7}$$

Note that if the point is outside the cube, we calculate the distance between the point and the nearest edges of the maps.

For the 3-dimensional case, there are four straight lines in the space. Therefore, the minimum value in four distances is chosen as the shortest distance D_1 for symbol "1". In the same way, D of symbol "0" is chosen as D_0. We calculate both of D_1 and D_0 for all l and find their summations $\sum D_1$ and $\sum D_0$. Finally, we decide the decoded symbol as 1 (or 0) for $\sum D_1 < \sum D_0$ (or $\sum D_1 > \sum D_0$).

The calculation of the shortest distance can be extended to N_d-dimensional space for $N_d \geq 4$.

III. Proposed Error-Correcting Method

Our proposed error-correcting method calculates the shortest distance by the double or triple of the sequence length N which used for the modulation in the transmitter, i.e. the double or triple dimension ($N_d = 2N$ or $3N$). Figure 4(a) shows the image of the operation of the error-correcting. To simplify the explanation, we assume the redundancy length to be $2N$. The redundancy is the sequence which has the

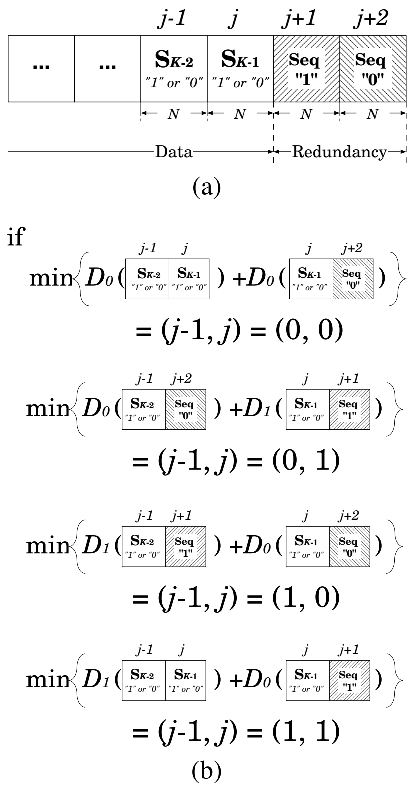


Fig. 4. Proposed Error-Correcting Method ($N_d = 2N$).

known information, “1” and “0”. Here, let us expect that the $j = (K-1)$th bit is “1” or “0”. Since the $(K-1)$th bit is “1” or “0” and the redundancy has “1” or “0” certainly, the $(K-1)$th bit sequence also has the relation with either of two types of the sequence based on the chaotic dynamics. Moreover the $(K-2)$th sequence also has the relation with either of three types of the sequence. Thus we can consider the j and $j-1$th bit as the four pattern, as shown in Fig 4(b), where $D_1(.)$ and $D_0(.)$ mean the shortest distance between the chaotic map of Symbol “1” and “0”, respectively. Since the sequence connected two types becomes the double sequence length, we can calculate the shortest distance using $N_d = 2N$-dimensional space. We calculate each distance like the pattern of Fig 4(b). Finally, we choose the pattern of the shortest distance out of the four patterns, and the $(K-1)$th bit is decided accurately. By using this determined symbol for detection of the next bit, these operation can be shifted and it can demodulate to all information. Therefore, the likelihood of information can improve by expecting the combination of the sequence based on the chaotic dynamics, and we can detect information in high accuracy. In this case, the coding rate becomes $\frac{K}{K+2}$.

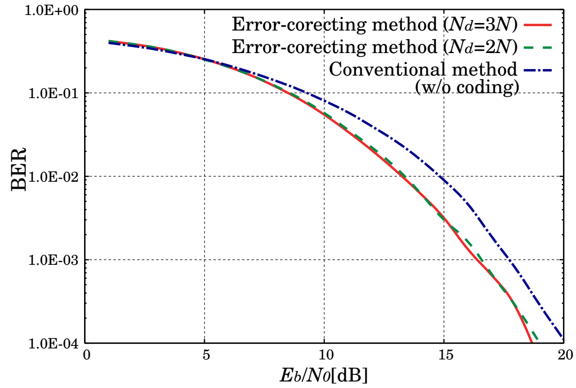
In the same way, we can calculate the shortest distance using $N_d = 3N$-dimensional space to perform the error-correcting. In this case, the redundancy length is $4N$, the combination of the sequence becomes the eight patterns and the coding rate becomes $\frac{K}{K+4}$.

IV. Simulation Results

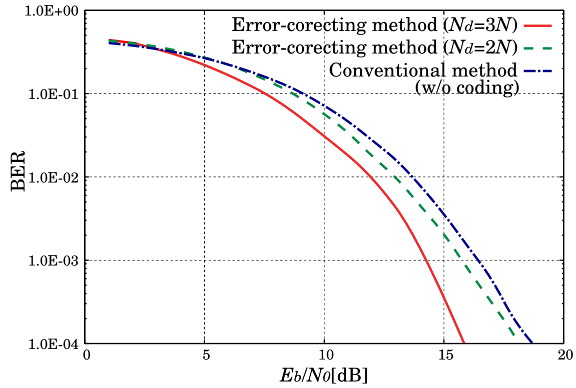
To evaluate the proposed error-correcting method, we carry out computer simulations with following simulation conditions. In the transmitting side, the packet consists of the data part with $K = 64$ bits and the redundancy part with $2N$ or $4N$. In this case, each coding rate becomes 0.96 or 0.94. Here, the parameter of the skew tent map is fixed as $a = 0.05$, and we use the chaotic sequence length $N = 3$ and 4. To compare the performance of N_d-dimensional space, we use $N_d = 2N$ and $N_d = 3N$. Based on these conditions, we iterate the simulation 10,000 times and calculate the average of BERs.

Figures 5(a) and (b) show the BERs versus E_b/N_0 for $N = 3$ and $N = 4$, respectively. To compare the performance of the error-correcting method, Figs. 5(a) and (b) also show the performance of our suboptimal receiver without the coding. From these results, we observe that the both BERs of the error-correcting method show gain over the system without the coding. Significant improvements are observed for $N = 4$ with $N_d = 3N$-dimensional. In this case, since the receiver calculate the shortest distance using $N_d = 12$-dimensional space, the computation complexity also increase. As our receiver calculates the likelihood of the received symbols not only by the shortest distance between the received signals and the chaotic map but also analyze the chaotic dynamics of the successive symbols, the likelihood of the received symbol increase and the performance improvement is achieved. Moreover, although the cording rate is very high, the proposed method can well correct the error. Therefore, we can say that the efficiency of the proposed error-coding is excellent.

However, we also confirm that the curve of the proposed method of $N = 3$ with $N_d = 3N$-dimensional space corresponds that with $N_d = 3N$-dimensional, as shown in Fig. 5(a).



(a) $N = 3$.



(b) $N = 4$.

Fig. 5. Simulation results.

We have not clearly understood the reason why two curves correspond, yet. We would like to investigate and analyze the chaotic sequence and its behavior in detail when our error-correcting method is used.

V. Conclusions

In this study, to improve the error performance of the suboptimal receiver more, we have proposed the error-correcting method using the features of chaos for the suboptimal receiver. As results, we have confirmed that the likelihood of the received symbol increases and obtained the 3dB gain in error performance.

Acknowledgments

This research is supported by the fund of Open Research Center Project from MEXT of Japanese Government (2007-2011).

References

[1] M. Hasler and T. Schimming, “Chaos communication over noisy channels,” *Int . Journal of Bifurcation and Chaos*, vol. 10, no. 4, pp. 719-736, Apr. 2000.
[2] F. C. M. Lau and C. K. Tse, *Chaos-Based Digital Communication Systems*, Springer, 2003.
[3] L. E. Larson, J-M. Liu, L. S. Tsimring, *Digital Communications Using Chaos and Nonlinear Dynamics*, Springer, 2006.
[4] S. Arai and Y. Nishio, “Detection of Information Symbols and Sequence Lengths Using Suboptimal Receiver for Chaos Shift Keying,” *Proc NOLTA’06*, pp. 799-802, Sep. 2006.